

An easy way to analyse octagonal slab

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ABSTRACT: This paper describes the FE analyses of octagonal slabs with a few selected support conditions. Based on the availability of middle strip, FE analyses described in this paper have been divided into two sets. In first set, where middle strip exists, middle strip moments and slab centre deflections have been compared with that of rectangular slab in order to identify the range of parameters which would allow treating an octagonal slab as a rectangular slab. In the second set, where middle strip does not exist, analyses have been performed to study octagonal slab behavior. Finally results of FE analyses have been utilized to formulate coefficient tables to compute the moment and deflection of octagonal slabs. Simplified method has been described where octagonal slab can be analyzed as a rectangular slab.

1 INTRODUCTION

In designing the rectangular slabs there are convenient design methods available to users to obtain the slab strip moments (ACI, 1963) and methods are also available to compute the deflection of such slabs (Ahmed and Chowdhury, 1999a, b). Such methods are not available for analysis and design of octagonal slabs. From the support conditions rectangular slabs are classified into nine categories (ACI, 1963). This number is dependent on the number of supporting edges of the slab. Considering the same Octagonal slabs can be classified in to a large number of categories. Instead of going through all the cases the paper aims to identify a few number of selected cases that are most likely to occur in building structures. In addition to that it is of importance to identify cases where a octagonal slab can be simplified as a rectangular slab and when it must be analysed as a octagonal slab. The present paper identifies such cases and also presents a tabular form of coefficients to compute the design moments and deflection of octagonal slabs with few selected support conditions.

2 SELECTION OF PARAMETERS FOR FE ANALYSES

To study the behavior of octagonal slabs in addition to the span ratio (a/b) (see figure 1); followings have been considered:

- Support conditions
- $a1/a$ and $b1/b$ ratios (see figure 1 for definition of $a1$, a , $b1$ and b)

2.1 *Support condition*

It is known that there are nine possible support conditions for rectangular slabs. Number of support cases increase rapidly with increasing number of slab edges. Thus it became impossible to cover up the full set of possible cases in the present research scheme. Only four support cases are considered in the present paper (Case A means all edges simply supported; Case B means all edges fixed; Case C means only one edge fixed while others simply supported and Case D means only one edge simply supported while others fixed as shown in Figure 1).

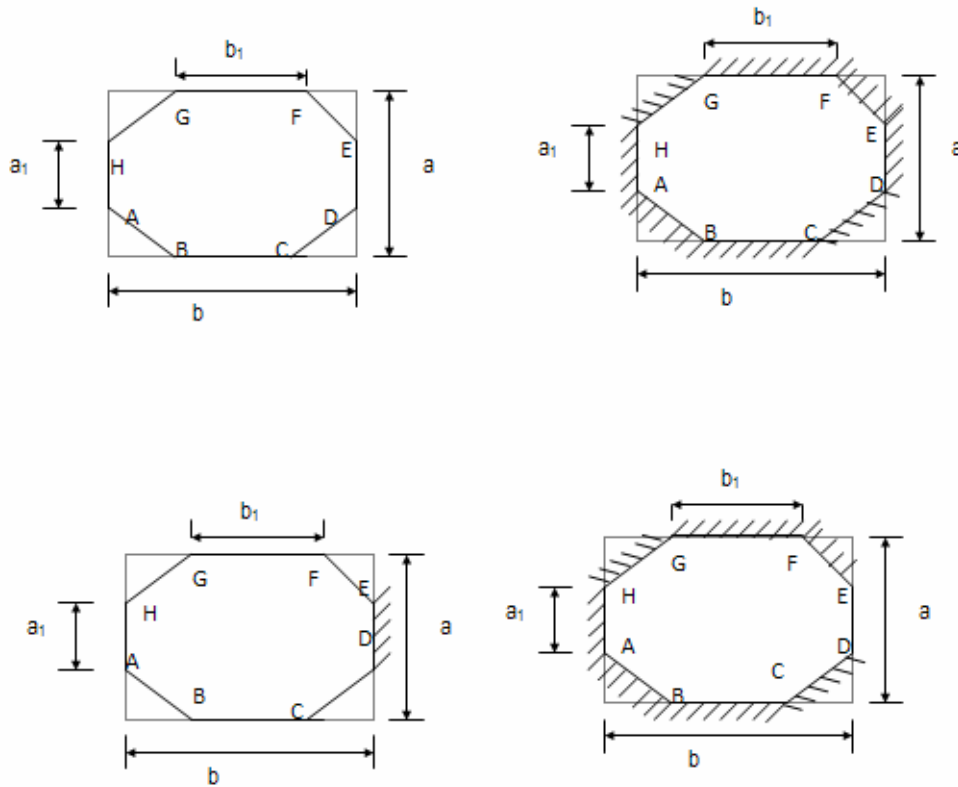


Figure 1: selected support conditions

2.2 a_1/a and b_1/b ratio

As mentioned in the introduction two sets of analyses have been selected. To conduct each set of analyses a certain range of parameters has been selected as shown in Table 1.

Table 1. Selected a_1/a and b_1/b ratios for set-1 and set-2

| Span ratio (a/b) | Set-1 | | Set-2 | |
|-------------------------|---------|---------|---------|---------|
| | a_1/a | b_1/b | a_1/a | b_1/b |
| 0.50 | 0.50 | 0.50 | 0.00 | 0.00 |
| | 0.75 | 0.75 | 0.25 | 0.25 |
| | 0.90 | 0.90 | 0.40 | 0.40 |
| 0.75 | 0.50 | 0.50 | 0.00 | 0.00 |
| | 0.75 | 0.75 | 0.25 | 0.25 |
| | 0.90 | 0.90 | 0.40 | 0.40 |
| 1.0 | 0.50 | 0.50 | 0.00 | 0.00 |
| | 0.75 | 0.75 | 0.25 | 0.25 |
| | 0.90 | 0.90 | 0.40 | 0.40 |

For octagonal slabs belonging to set-1 middle strip moment and slab centre deflection to be compared with that of the rectangular slab. Using results of set-1 and that of set-2 coefficients can be obtained so that moment and deflection can be calculated by using these coefficients using simplified equations.

3 COMPUTATION OF DESIGN MOMENTS FROM FE RESULTS

To compute the middle strip moments of octagonal slabs belonging to the first set (set-1) using FE results method described by Ahmed and Chowdhury (1999a, b) have been followed. To calculate moments of octagonal slabs belonging to the second set (set-2) from the results of EF model, since middle strip ($a/2$ or $b/2$) is not available, different approach is essential that is described below.

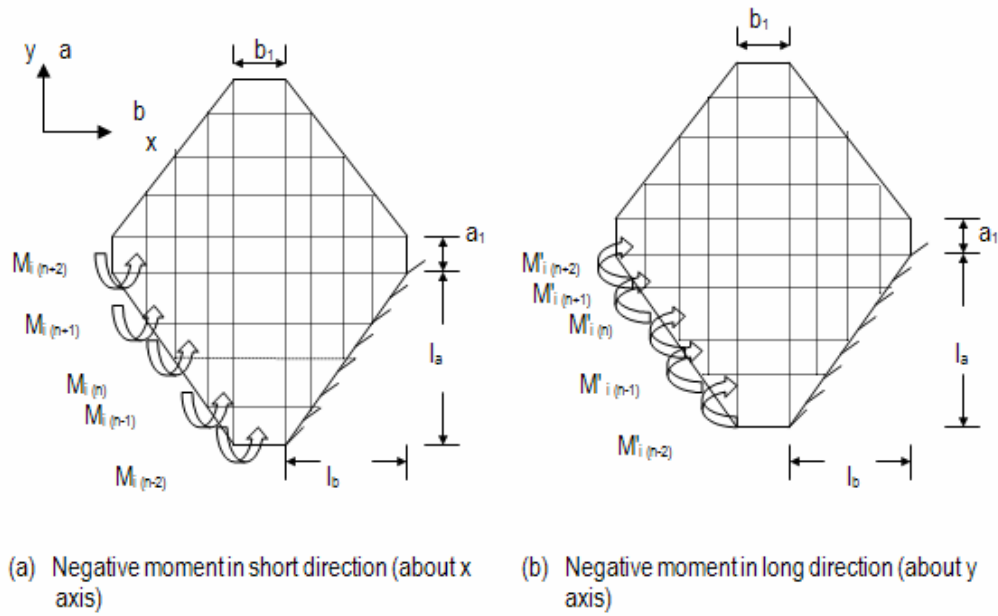


Figure 2: Calculation of moments in the inclined portion of octagonal slab (continued)

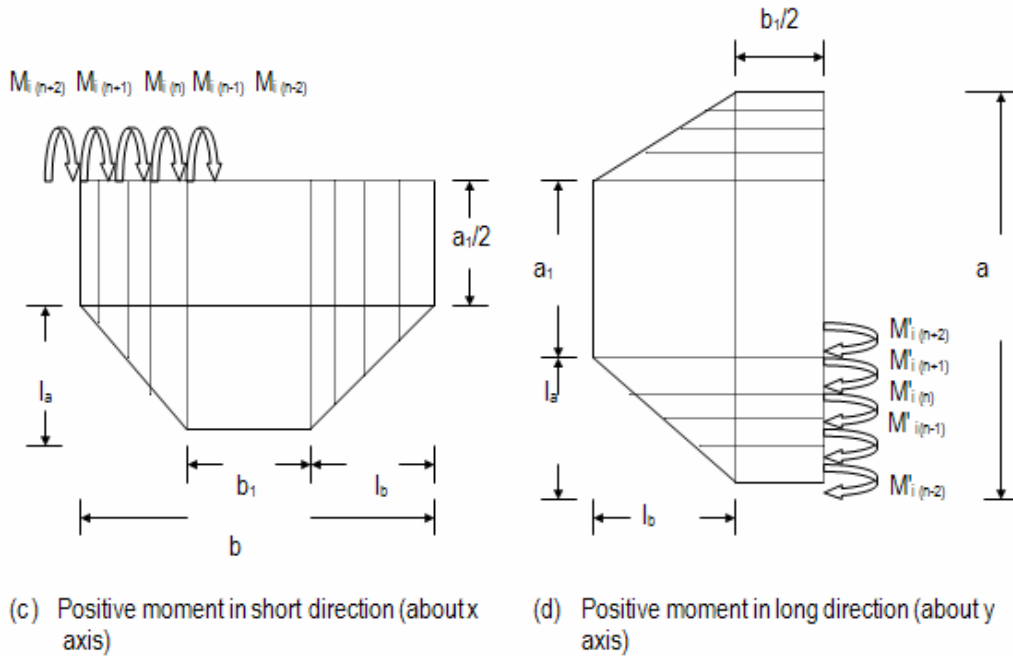


Figure 2: Calculation of moments in the inclined portion of octagonal slab

Figure 2 shows the procedure to calculate moment at inclined portion of octagonal slab. Moments $M_{i(n-2)}$, $M_{i(n-1)}$, $M_{i(n)}$, $M_{i(n+1)}$, $M_{i(n+2)}$ and $M'_{i(n-2)}$, $M'_{i(n-1)}$, $M'_{i(n)}$, $M'_{i(n+1)}$, $M'_{i(n+2)}$ in figures 2(a), 2(b), 2(c), 2(d) are calculated using the method to convert of nodal moments to slab internal moment shown in Ahmed and Chowdhury (1999a,b). From figure 2 (a), negative support moment in short direction (M_{ia}) can be obtained by averaging them over the horizontal portion (l_b) of the inclined part of octagonal slab, as the FE results are per unit width of element. Thus approximately,

$$M_{ia}^- = \frac{M_{i(n-2)} + M_{i(n-1)} + M_{i(n)} + M_{i(n+1)} + M_{i(n+2)}}{l_b} \quad (1)$$

Similarly, as shown in Figure 2 (b), support moment in long direction (M_{ib}^-) can be obtained by averaging them over the horizontal portion (l_a) of the inclined part of octagonal slab. Thus, approximately,

$$M_{ib}^- = \frac{M'_{i(n-2)} + M'_{i(n-1)} + M'_{i(n)} + M'_{i(n+1)} + M'_{i(n+2)}}{l_a} \quad (2)$$

Similarly, from figure 2 (c),

$$M_{ia}^+ = \frac{M_{i(n-2)} + M_{i(n-1)} + M_{i(n)} + M_{i(n+1)} + M_{i(n+2)}}{l_b} \quad (3)$$

from figure 2 (d),

$$M_{ib}^+ = \frac{M'_{i(n-2)} + M'_{i(n-1)} + M'_{i(n)} + M'_{i(n+1)} + M'_{i(n+2)}}{l_a} \quad (4)$$

For straight portion of octagonal slab, positive moment in short direction can be obtained by averaging them over the straight portion (b_1). Thus, from figure 3 (a),

$$M_{sa}^+ = \frac{M_{s(n-2)} + M_{s(n-1)} + M_{s(n)} + M_{s(n+1)} + M_{s(n+2)}}{b_1} \quad (5)$$

From figure 3(b),

$$M_{sb}^+ = \frac{M'_{s(n-2)} + M'_{s(n-1)} + M'_{s(n)} + M'_{s(n+1)} + M'_{s(n+2)}}{a_1} \quad (6)$$

Similar method is applied for evaluation of the edge moments of straight portion.

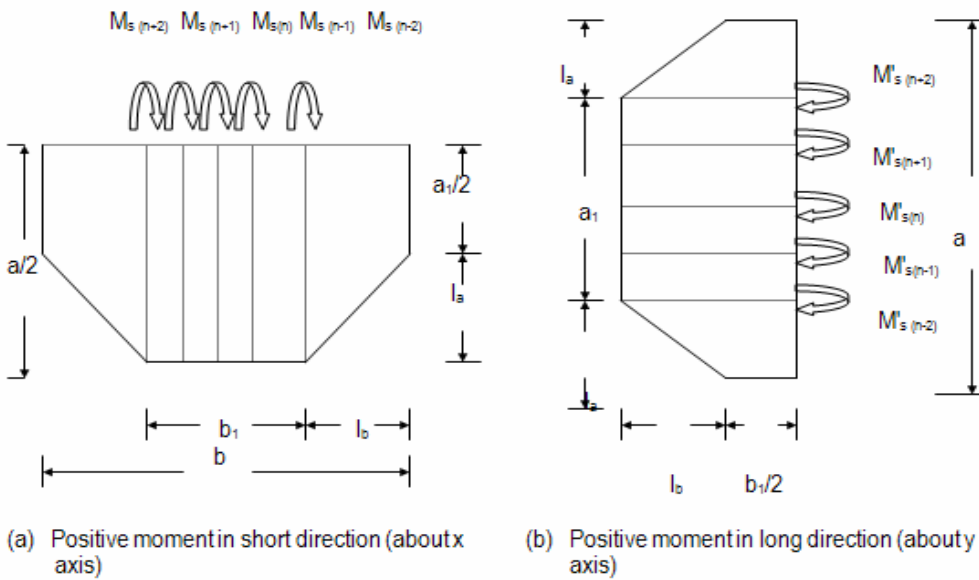


Figure 3: Calculation of moments of straight portion of octagonal slab

Instead of computing moments, it is desirable to compute moment coefficients. It is possible to compute the coefficients for differential cases and span ratios, as follows:

$$M_{ia,FE}^- = C_{ia,FE}^- W l_{ea}^2 \quad (7)$$

$$C_{ia,FE}^- = \frac{M_{ia,FE}^-}{W l_{ea}^2} \quad (8)$$

Where $l_{ea} = \sqrt{\frac{1}{6}(a_1^2 + 2l_{2a}^2 + 2l_{3a}^2 + a^2)}$ (see figure 4), and W = Total uniformly distributed load

Similarly, C_{ib}^- , C_{ia}^+ , C_{ib}^+ can be obtained.

For straight portion of octagonal slab,

$$M_{sa,FE}^+ = C_{sa,FE}^+ W a^2 \quad (9)$$

$$C_{sa,FE}^+ = \frac{M_{sa,FE}^+}{W a^2} \quad (10)$$

where a = short span of slab

Similarly, C_{sb}^+ , C_{sa}^- , C_{sb}^- can be computed.

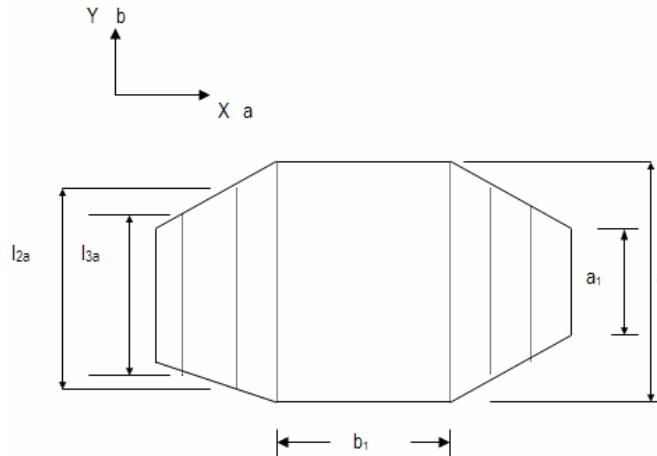


Figure 4: Calculation of equivalent length for calculating the moment of inclined portion of octagonal slab

4 RESULT OF FE ANALYSES

4.1 Verification of FE model with selected case

As extensive literature on octagonal slabs is not available, it is not possible directly verify results of FE model for such slabs. But for the purpose of checking the accuracy of introducing triangular elements, yet some verification is essential. This is conducted by comparing results of rectangular slabs from Chowdhury (2000), indicated by case a, with results of mesh described above for cases $a_1/a = b_1/b = 0.00$ (indicated by case b) and $a_1/a = b_1/b = 0.75$, indicated by case c, for various a/b ratios, as these a_1/a and b_1/b in the two later cases closely represents or nearly represents rectangular slab system. Thus these should produce same or very close displacement at the centre when subjected to same load. Since deflection is to be same, the coefficient of deflection should also be same. So comparison of slab centre deflection coefficient should be useful for verification of the new FE models which is shown in figure 5. Besides this the moments should be also same or very close and the moment coefficients can also be used for the purpose of verification (results compared well described in Chowdhury (2000)).

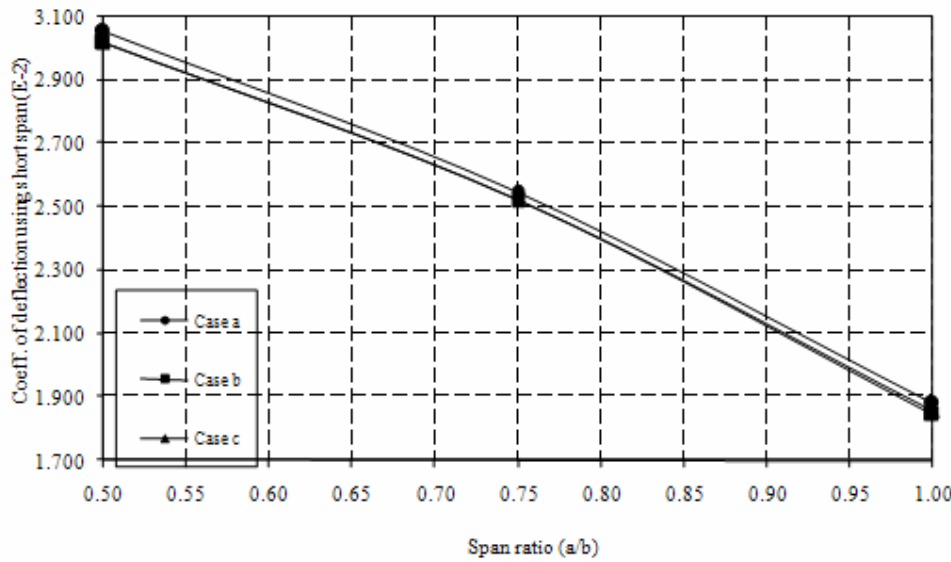


Figure 5: Comparison of deflection of meshes obtained from different cases for support case D

4.2 Effect of a_1/a and b_1/b ratio on middle strip moment

Figure 6 shows the variation of positive moment coefficient ratio in short and long direction with b_1/b ratio for different a_1/a ratios for span ratio (a/b) 0.50. Moment coefficient ratio for different selected support cases and a/b ratios has been obtained by dividing the middle strip moment coefficient (positive or negative) in short or long direction of octagonal slab of given a_1/a , b_1/b ratio by the same of rectangular slab in the respective direction; both having same a/b ratio and support condition. It has been observed from Figure 6 and the results of analyses Chowdhury (2000) that the moment coefficient ratio (positive and negative) is either constant or increases with a_1/a and b_1/b ratio for all support conditions. It indicates that middle strip moment of octagonal slab is either equal or increases with a_1/a and b_1/b ratio than that of rectangular slab for any support condition. This is happening because load distribution is either constant or increases with a_1/a and b_1/b ratio. From figure 7, It can also be seen that the ratio of positive moment coefficient in long direction is either constant or decreases with a_1/a and b_1/b ratio for all support conditions as load distribution is either constant or decreases. From figures 6, 7 and other results of analyses Chowdhury(2000), It can be concluded that maximum increase in long direction positive and negative moment is 30% and 15% respectively and maximum decrease in long and short direction negative moment is 60% and 25% respectively, with respect to rectangular slab.

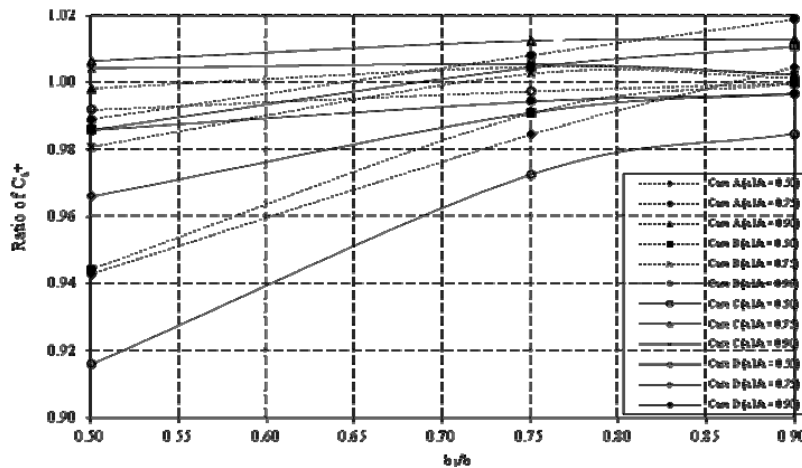


Figure 6: Variation of C_a^+ ratio with b_1/b ratio for a/b ratio 0.50

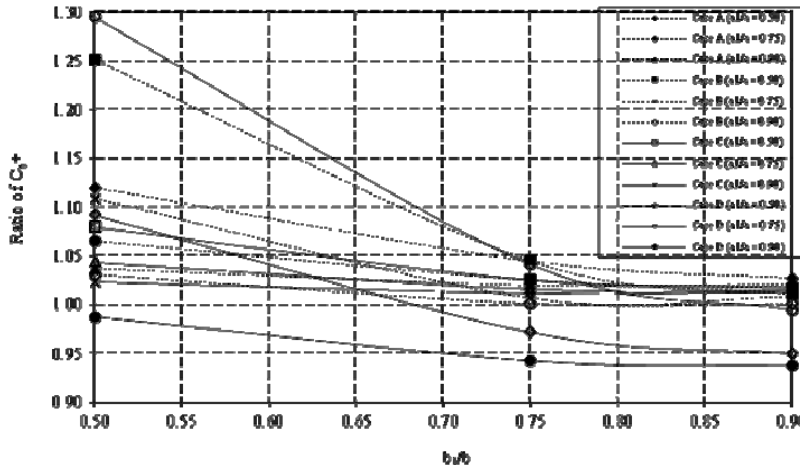


Figure 7: Variation of C_b^+ ratio with b_1/b ratio for a/b ratio 0.50

4.3 Effect of a_1/a and b_1/b ratio on moment of straight portion of the octagonal slab

Analyses conducted by Chowdhury (2000) shows that variation of C_{sb}^+ for different span ratios shows that C_{sb}^+ increases gradually when b_1/b ratio varies from 0.00 to 0.25 but after that it does not change considering all the selected support cases and a/b ratios. Variation of C_{sa}^+ for different span ratios shows that C_{sa}^+ is almost constant with b_1/b ratio. Variation of C_{sa}^- , C_{sb}^- for different span ratios, shows that C_{sa}^- , C_{sb}^- increases sharply with b_1/b ratio considering all the selected support cases. Variation of C_{sa}^+ , C_{sb}^+ , C_{sa}^- , and C_{sb}^- with a_1/a ratio increases. This is occurred due to the fact that all the moment coefficients increases as load distribution of the straight portion increases with a_1/a and b_1/b ratio. It is not possible to obtain the variation of C_{sa}^+ , C_{sa}^- for cases A, B, C and D of b_1/b ratio = 0.00 with a_1/a ratio as the straight portion of octagonal slab is not available. For the same reasons the variation of C_{sb}^+ , C_{sb}^- for cases A, B, C and D of $a_1/a = 0.0$ with b_1/b ratio is not possible to obtain. Results presented by Chowdhury [4] can be utilised for computing the moment of straight portion of octagonal slab by the help of equation 9.

4.4 Effect of a_1/a and b_1/b ratio on moment of inclined portion of the octagonal slab

Variation of C_{ia}^+ , C_{ia}^- , C_{ib}^+ , C_{ib}^- for different span ratios shows that C_{ia}^+ , C_{ia}^- , C_{ib}^+ decreases with a_1/a ratio whereas coefficient of negative moment of inclined portion of octagonal slab (C_{ib}^-) in long direction increases with a_1/a ratio. Results presented by Chowdhury (2000) can be utilised for computing the moment of inclined portion of octagonal slab by the help of equation (7).

4.5 Effect of a_1/a and b_1/b ratio on deflection

Figure 8 shows the variation of deflection ratio with b_1/b ratio for different a_1/a ratios, selected support cases and 0.50 span ratio. Here deflection ratio has been defined for different selected support cases and span ratios, by dividing the slab centre deflection of octagonal slab of given a_1/a , b_1/b ratio by the same of rectangular slab considering same span ratio, support cases. Considering deflection of numerical analysis set-1, slab can be analysed as rectangular slab since maximum 10% variation of deflection have been observed. Slab centre deflection of octagonal slab set-1 can be directly computed knowing the slab centre deflection of rectangular slab using results presented by Chowdhury (2000). From results presented by Chowdhury (2000) for set-1 and set-2 it can be seen that deflection ratio increases with a_1/a and b_1/b ratio as load distribution increases with a_1/a and b_1/b ratio.

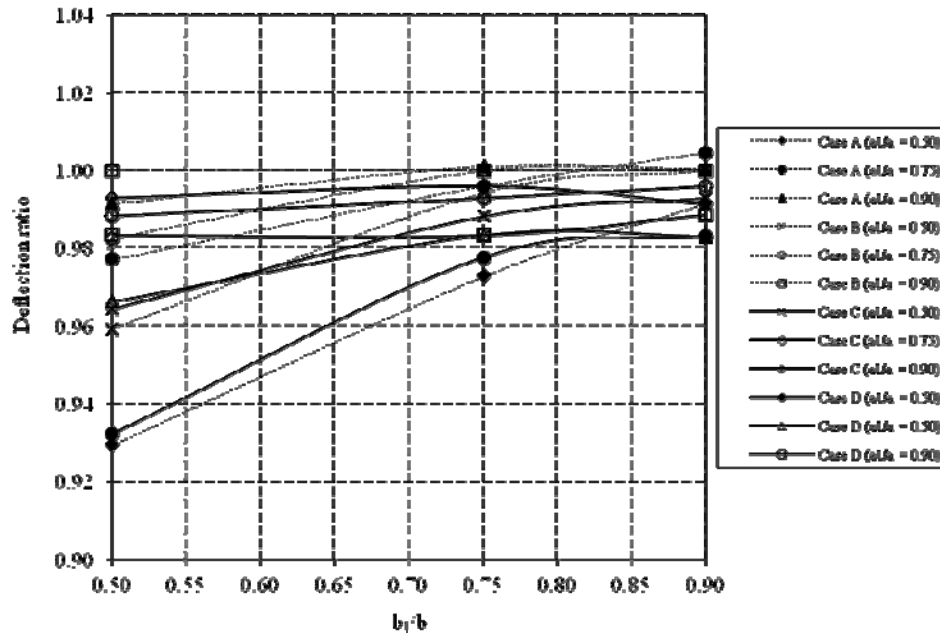


Figure 8: Variation of deflection ratio with b_1/b ratio for a/b ratio 0.50

5 DESIGN CONSIDERATIONS

Tables 2(a) and 2(b) list the range of parameters when the positive and negative moments in long direction are significantly higher than that of rectangular slabs, as obtained from the FE analyses.

Table 2(a) Range of parameters when positive moments in long direction are significantly greater than that of rectangular slab

| Support case | a_1/a | b_1/b | Span ratio (a/b) |
|--------------|--------------|--------------|----------------------|
| A | ≤ 0.50 | 0.50 to 0.70 | ≤ 0.50 |
| B | ≤ 0.50 | 0.50 to 0.90 | 0.50 to 0.75 |
| D | 0.50 to 0.75 | 0.50 to 0.90 | 0.50 to 0.75 |

Table 2 (b) Range of parameters when negative moments in long direction are significantly greater than that of rectangular slabs

| Support case | a_1/a | b_1/b | Span ratio (a/b) |
|--------------|-------------|--------------|----------------------|
| C | ≤ 0.50 | 0.70 to 0.90 | 0.50 to 0.75 |

Tables 2(c) and 2(d) list the range of parameters when negative moments in long and short direction are significantly lower than that of rectangular slabs respectively.

Table 2 (c) Range of parameters when negative moments in long direction are significantly lower than that of rectangular slab

| Support case | a_1/a | b_1/b | Span ratio (a/b) |
|--------------|--------------|--------------|----------------------|
| B | 0.50 to 0.90 | 0.50 to 0.90 | 0.50 to 0.75 |
| | ≤ 0.50 | 0.50 to 0.90 | ≤ 1.0 |
| D | 0.50 to 0.90 | 0.50 to 0.90 | ≤ 0.50 |
| | ≤ 0.50 | 0.50 to 0.90 | 0.75 to 1.0 |

Table 2(d) Range of parameters when negative moments in short direction are significantly lower than that of rectangular slab

| Support case | a_1/a | b_1/b | Span ratio (a/b) |
|--------------|---------|--------------|------------------|
| B | 0.50 | 0.50 to 0.75 | 0.75 to 1.0 |
| D | 0.50 | 0.50 to 0.90 | 0.50 to 1.0 |

So octagonal slab with the range of parameters shown in Tables 2a, 2b, 2c and 2d must be analysed as octagonal slab. Table 3 lists the range of parameters when octagonal slab can be analysed as rectangular slab with maximum variation in moment within a range of 10% with respect to rectangular slab.

Table 3 Range of parameters when octagonal slab can be simplified as rectangular slab

| Support case | a_1/a | b_1/b | Span ratio (a/b) |
|--------------|--------------|--------------|------------------|
| A | ≤ 0.50 | 0.71 to 0.90 | ≤ 0.50 |
| | 0.75 to 0.90 | 0.50 to 0.90 | ≤ 0.50 |
| | 0.50 to 0.90 | 0.50 to 0.90 | 0.75 to 1.0 |
| B | 0.75 to 0.90 | 0.50 to 0.90 | ≤ 1.0 |
| C | ≤ 0.50 | 0.50 to 0.69 | 0.50 to 0.75 |
| | 0.75 to 0.90 | 0.50 to 0.90 | 0.50 to 0.75 |
| | 0.50 to 0.90 | 0.50 to 0.90 | ≤ 1.0 |
| D | 0.75 to 0.90 | 0.50 to 0.90 | 0.75 to 1.0 |

Middle strip moment of such octagonal slab can be directly computed knowing the middle strip moment of rectangular slab, if the slabs are within the above limits. Finally using results of analysis, moment coefficient tables have been formed in Chowdhury (2000) so that middle strip moments and deflection can be computed directly.

6 CONCLUSIONS

The paper described the finite element analysis of octagonal slabs with four support cases are considered (all edges simply supported; all edges fixed; only one edge fixed while others simply supported; only one edge simply supported while others fixed). Equations have been formulated to obtain strip moments for such slabs from ANSYS results that are in terms of moment at a node per element. From the results of FE analysis it has been possible to identify when it is possible to analyze a slab as a rectangular slab. Besides this a set of coefficient tables have been prepared (presented in Chowdhury (2000) using results of FE analyses, that can be used to analyse only octagonal slab with mentioned support conditions.

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